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SCHUR MULTIPLICATORS OF INFINITE PRO-p-GROUPS WITH FINITE COCLASS

BY

Bettina Eick

Institut Computational Mathematics, TU Braunschweig, Pockelsstrasse 14, 38106 Braunschweig, Germany
e-mail: beick@tu-bs.de

ABSTRACT

Let G be an infinite pro-p-group of finite coclass and let $M(G)$ be its Schur multiplicator. For $p > 2$, we determine the isomorphism type of $\mathrm{Hom}(M(G), \mathbb{Z}_p)$, where $\mathbb{Z}_p$ denotes the p-adic integers, and show that $M(G)$ is infinite. For $p = 2$, we investigate the Schur multiplicators of the infinite pro-2-groups of small coclass and show that $M(G)$ can be infinite, finite or even trivial.

1. Introduction

The infinite pro-p-groups of finite coclass play a central role in the classification and investigation of finite p-groups by coclass. In particular, as shown in [3], the Schur multiplicators of the infinite pro-p-groups of coclass r have a significant influence on the Schur multiplicators of the finite p-groups of coclass r so that it is of interest to determine the infinite pro-p-groups of finite coclass with finite or even trivial Schur multiplicator. It is the aim of this paper to investigate this problem.

Let G be an infinite pro-p-group of finite coclass. Consider the series $G \geq C > T > N$, where N is the hypercenter of G, the factor T/N is the Fitting subgroup of G/N, and C/T is the center of G/T. Then C/T is cyclic of order p^t for some $t \geq 1$. (See Section 2 for background.) We call t the **central exponent** of G.

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The Schur multiplicator $M(G)$ of an infinite pro-p-group of finite coclass is defined as $M(G) = H_2(G, \mathbb{Z}_p)$, where $\mathbb{Z}_p$ denotes the p-adic integers. The pro-p-group $M(G)$ is abelian of finite rank and hence has the form $M(G) = T(G) \times F(G)$, where $T(G)$ is a finite p-group and $F(G) \cong \mathbb{Z}_p^l$ for some $l \in \mathbb{N}_0$. As $\mathrm{Hom}(M(G), \mathbb{Z}_p) \cong \mathbb{Z}_p^l$, it follows that l can also be characterised as the rank of $\mathrm{Hom}(M(G), \mathbb{Z}_p)$. We call l the **torsion-free rank** of $M(G)$ and denote it by $\mathrm{tf}(M(G))$.

The central aim of this paper is to prove the following theorem.

Theorem A: *Let $p > 2$ and let G be an infinite pro-p-group of finite coclass with central exponent t. Then $\mathrm{tf}(M(G)) = p^{t-1}(p-1)/2$ and thus $M(G)$ is infinite.*

This theorem is not valid for $p = 2$. For example, the infinite pro-2-group

$$\langle a, t \mid a^{2^r} = 1, t^a = t^{-1} \rangle \cong \mathbb{Z}_2 \rtimes C_{2^r}$$

has coclass r and trivial Schur multiplicator. We include a list of the Schur multiplicators of all infinite pro-2-groups of coclass at most 3 below. (See Section 7.)

2. Preliminaries

In this section we briefly recall the well-known structure of infinite pro-p-groups of finite coclass and we outline various details on the series introduced in Section 1.

1 Theorem: *Let $p > 2$ and let G be an infinite pro-p-group of coclass r with series $G \geq C > T > N$ as defined in Section 1. Then*

(a) *N is finite of order p^m for some $m < r$ and G/N has coclass $r - m$.*
(b) *$T/N \cong \mathbb{Z}_p^d$ with $d = p^{s-1}(p-1)$ for some $s \in \{1, \ldots, r-m\}$.*
(c) *G/T is a finite non-trivial p-group which embeds into $\mathrm{GL}(d, \mathbb{Z}_p)$.*
(d) *C/T is cyclic of order p^t for some $t \in \{1, \ldots, s\}$.*

Proof. (a) See [4], Lemma 7.4.4.

(b)+(c) See [4], Theorem 7.4.12.

(d) As G/T is a finite non-trivial p-group, its center C/T is non-trivial. By [4], Theorem 7.4.12, The group G/T acts irreducibly on $T \otimes \mathbb{Q}_p$ and hence has a cyclic centre. As the maximal dimension of an irreducible faithful $\mathbb{Q}_p$-representation of the cyclic group of order p^t has dimension $p^{t-1}(p-1)$, it follows that $1 \leq t \leq s$. ∎

3. A cohomological characterisation

As a first step towards our aim, we characterise the torsion-free rank of the Schur multiplicator of an infinite pro-p-group with finite coclass using cohomology.

2 Theorem: *Let G be a pro-p-group of finite coclass. Then* $\mathrm{tf}(M(G)) = \mathrm{tf}(H^2(G, \mathbb{Z}_p))$.

Proof. The Universal Coefficients Theorem (see [6], page 349) yields that $H^2(G, \mathbb{Z}_p)$ is an extension of $\mathrm{Ext}(G/G', \mathbb{Z}_p)$ by $\mathrm{Hom}(M(G), \mathbb{Z}_p)$. As G/G' is finite, it follows that $\mathrm{Ext}(G/G', \mathbb{Z}_p)$ is finite and hence the result follows. ∎

4. Finite extensions and subgroups of finite index

The following theorem reduces the proof of the main theorem of this paper to the case of infinite pro-p-groups with finite coclass and trivial hypercenter.

3 Theorem: *Let G be an infinite pro-p-group of finite coclass and let L be a finite central subgroup in G. Then* $\mathrm{tf}(M(G)) = \mathrm{tf}(M(G/L))$.

Proof. We consider the 5-term homology and cohomology sequences (see [4], Corollary 9.4.12). These imply the following two exact sequences:

$$M(G) \to M(G/L) \to L \quad \text{and} \quad H^1(L, \mathbb{Z}_p)^G \to H^2(G/L, \mathbb{Z}_p) \to H^2(G, \mathbb{Z}_p).$$

The first sequence yields that $\mathrm{tf}(M(G)) \geq \mathrm{tf}(M(G/L))$, since L is finite. The second sequence and Theorem 2 imply that $\mathrm{tf}(M(G)) \leq \mathrm{tf}(M(G/L))$, since $H^1(L, \mathbb{Z}_p)$ is finite. In summary, we obtain that $\mathrm{tf}(M(G)) = \mathrm{tf}(M(G/L))$ as desired. ∎

The next theorem considers normal subgroups of finite index of infinite pro-p-groups with finite coclass. It yields an important tool towards the proof of the main theorem of this paper.

4 Theorem: *Let G be an infinite pro-p-group of finite coclass and let N be a normal subgroup of finite index in G. Then G/N acts naturally on $M(N)$ with image $M(N)^{G/N}$ and* $\mathrm{tf}(M(G)) = \mathrm{tf}(M(N)/M(N)^{G/N})$.

Proof. We consider the Lyndon–Hochschild–Serre spectral sequence for $H_2(G,\mathbb{Z}_p)$ using N as normal subgroup; We refer to [9] for background. The E^2-terms in this sequence are $E^2_{pq} = H_p(G/N, H_q(N,\mathbb{Z}_p))$. As G/N is finite, it follows that E^2_{pq} is finite for all $p > 0$. This implies that E^r_{pq} is finite for all $p > 0$ and $r \geq 2$ and it yields that $\mathrm{tf}(E^r_{0q}) = \mathrm{tf}(E^2_{0q})$ for all q and $r \geq 2$. Hence we obtain that $\mathrm{tf}(H_2(G,\mathbb{Z}_p)) = \mathrm{tf}(E^\infty_{02}) = \mathrm{tf}(E^2_{02}) = \mathrm{tf}(H_0(G/N, H_2(N,\mathbb{Z}_p)) = \mathrm{tf}(M(N)/M(N)^{G/N})$ as desired. ∎

5. Space groups

Let G be an infinite pro-p-group with finite coclass and trivial hypercenter. Then G is an extension of a maximal abelian normal subgroup T by a finite p-group P which acts faithfully on T. Thus G has the structure of a space group with translation subgroup T and point group P.

Let $V = T \otimes \mathbb{Q}_p$. Then V and its tensor square $V \otimes V$ are natural modules for P. The module $V \otimes V$ splits as $\mathbb{Q}_pP$-module into a symmetric and an antisymmetric part which we denote by $V \otimes V = (V \vee V) \oplus (V \wedge V)$. The subspace of fixed points under the action of P in $V \wedge V$ is denoted by $\mathrm{Fix}_P(V \wedge V)$.

5 Theorem: *Let G be an infinite pro-p-group with finite coclass and trivial hypercenter. Let P be its point group, T its translation subgroup and $V = T \otimes \mathbb{Q}_p$. Then* $\mathrm{tf}(M(G)) = \dim(\mathrm{Fix}_P(V \wedge V))$.

Proof. Note that $M(T) \cong T \wedge T$ as P-module and thus $\mathrm{tf}(M(T)/M(T)^P) = \dim((V \wedge V)/(V \wedge V)^P) = \dim(\mathrm{Fix}_P(V \wedge V))$, since $V \wedge V$ is semisimple as P-module. Now the result follows directly from Theorem 4. ∎

Next, we investigate the fixed points $\mathrm{Fix}_P(V \wedge V)$ in more detail. The following theorem together with Theorems 5 and 3 complete the proof for the main Theorem A of this paper.

6 Theorem: *Let $p > 2$ and let P be the point group of an infinite pro-p-group G with finite coclass, trivial hypercenter and central exponent t. Then* $\dim(\mathrm{Fix}_P(V \wedge V)) = p^{t-1}(p-1)/2$.

Proof. Let $d_s = p^{s-1}(p-1)$ be the dimension of G and let $q = p^{s-t} = d_s/d_t$ with $d_t = p^{t-1}(p-1)$. Let $U_t(s) \cong C_{p^t} \wr P_{s-t} \leq \mathrm{GL}(d_s, \mathbb{Q}_p)$, where C_{p^t} is cyclic of order p^t and P_{s-t} is a Sylow p-group subgroup of $\mathrm{Sym}(q)$. Let C denote the center of P. Then by [2], Theorems 17 and 19, it follows that P is conjugate to a subgroup of $U_t(s)$ such that C conjugates to the center of $U_t(s)$ and thus to the diagonal subgroup of the base group of the wreath product. We assume in the following that $P \leq U_t(s)$ with $C = Z(P) = Z(U_t(s))$. Let $\overline{\mathbb{Q}}_p$ be the algebraic closure of $\mathbb{Q}_p$ and let $\overline{V} = V \otimes \overline{\mathbb{Q}}_p$.

The the character φ of C on $\overline{V}$ has the form $\varphi = q(\varphi_1 + \cdots + \varphi_{d_t})$, where $\varphi_1, \ldots, \varphi_{d_t}$ are the d_t different faithful characters of C over $\overline{\mathbb{Q}}_p$. This yields that $\varphi^2 = q^2 \sum_{1\leq i,j\leq d_t} \varphi_i\varphi_j$. Note that for every $i \in \{1, \ldots, d_t\}$ there exists a unique $j \neq i$ with $\overline{\varphi_j} = \varphi_i$. Thus

$$[\varphi^2, 1] = q^2 \sum_{1\leq i,j\leq d_t} [\varphi_i\varphi_j, 1] = q^2 \sum_{1\leq i,j\leq d_t} [\varphi_i, \overline{\varphi_j}] = q^2 d_t$$

which yields that $\dim(\mathrm{Fix}_C(V \otimes V)) = q^2 d_t$. Further, $\varphi \wedge \varphi = \sum_{1\leq i<j\leq d_t} \varphi_i\varphi_j$ and hence

$$[\varphi \wedge \varphi, 1] = q^2 \sum_{1\leq i<j\leq d_t} [\varphi_i\varphi_j, 1] = q^2 \sum_{1\leq i<j\leq d_t} [\varphi_i, \overline{\varphi_j}] = q^2 d_t/2$$

which yields that $\dim(\mathrm{Fix}_C(V \wedge V)) = q^2 d_t/2$.

The character χ of P on $\overline{V}$ has the form $\chi = \chi_1 + \cdots + \chi_{d_t}$, where χ_i is irreducible with $\chi_i|_C = q\varphi_i$. Let $\overline{V} = \overline{V}_1 \oplus \cdots \oplus \overline{V}_{d_t}$ be the corresponding decomposition of $\overline{V}$ into irreducible modules. We fix $i \in \{1, \ldots, d_t\}$ and investigate the action of P on $W = \overline{V}_i$. Let w be a p^t-th primitive root of unity in $\overline{Q}_p$. As $P \leq U_t(s)$, it follows that every element g of P acts as $g = ah$ on W, where a is a diagonal matrix with diagonal entries $w^{a_1}, \ldots, w^{a_q}$ for certain $a_1, \ldots, a_q$ and h is a permutation matrix corresponding to some $\pi \in \mathrm{Sym}(q)$. This allows one to determine $\chi_i(g)$ explicitly as $\chi_i(g) = \mathrm{trace}(ah) = w^{a_{f_1}} + \cdots + w^{a_{f_l}}$, where $f_1, \ldots, f_l$ are the fixed points of π. We note that $\chi_i(g^{-1}) = \mathrm{trace}((ah)^{-1}) = \mathrm{trace}((a^{-1})^h h^{-1}) = w^{-a_{f_1}} + \cdots + w^{-a_{f_l}}$. Thus $\overline{\chi_i} = \chi_j$ if and only if $\overline{\varphi_i} = \varphi_j$. This implies that

$$[\chi^2, 1] = \sum_{1\leq i,j\leq d_t} [\chi_i\chi_j, 1] = \sum_{1\leq i,j\leq d_t} [\chi_i, \overline{\chi_j}] = d_t$$

which yields that $\dim(\mathrm{Fix}_P(V \otimes V)) = d_t$. Similarly,

$$[\chi \wedge \chi, 1] = \sum_{1\leq i<j\leq d_t} [\chi_i\chi_j, 1] = \sum_{1\leq i<j\leq d_t} [\chi_i, \overline{\chi_j}] = d_t/2$$

and thus we obtain that $\dim(\mathrm{Fix}_P(V \wedge V)) = d_t/2$. ∎

We note that Theorem 6 is wrong for $p = 2$ as our examples in Section 7 show.

6. An example

Hopf's formula can be used to determine the Schur multiplicator of an explicitly given infinite pro-p-group of finite coclass. We use this feature to determine the Schur multiplicators of the infinite pro-p-groups $C_t = \mathbb{Z}_p^{d_t} \rtimes C_{p^t}$, where the cyclic group C_{p^t} acts uniserially on $\mathbb{Z}_p^{d_t}$ and $d_t = p^{t-1}(p-1)$.

7 Theorem: *$M(C_t) \cong \mathbb{Z}_p^{d_t/2}$ holds, unless $p = 2$ and $t = 1$ in which case $M(C_t) = \{1\}$.*

Proof. Let $d = d_t$ and $q = p^t$ to shorten notation. For $i \in \mathbb{Z}$ let $e_i = 1$ if $p^{t-1} \mid i-1$ and $e_i = 0$ otherwise. Then C_t has a finite presentation F/R, where F is the free group on the generators $\{g, t_1, \ldots, t_d\}$ and R is generated by the relations

$$g^q = 1,\ t_1^g = t_d^{-1},\ t_i^g = t_{i-1}t_d^{-e_i}\ (1 < i \leq d),\ t_i^{t_j} = t_i\ (1 \leq j < i \leq d).$$

We define the elements $x, x_{ji} \in F$ for $0 \leq j < i \leq d$ by

$$\begin{aligned} x &:= g^q, \\ x_{01} &:= t_1^g / t_d^{-1}, \\ x_{0i} &:= t_i^g / (t_{i-1}t_d^{-e_i}) \text{ for } 2 \leq i \leq d, \text{ and} \\ x_{ji} &:= t_i^{t_j} / t_i \text{ for } 1 \leq j < i \leq d. \end{aligned}$$

Then $R/[R,F]$ is generated by $\overline{x} := x[R,F]$ and $\overline{x}_{ji} := x_{ji}[R,F]$ for $0 \leq j < i \leq d$. To determine the isomorphism type of $R/[R,F]$, we determine the relations among the elements $\overline{x}$ and $\overline{x}_{ji}$. The 'confluence test' as described in [7], Chapter 9.8, yields that it is sufficient to determine the relations arising from the following list of equations

$$t_i(t_jt_k) = (t_it_j)t_k \text{ for } k < j < i \tag{1}$$

$$t_i(t_jg) = (t_it_j)g \text{ for } j < i \tag{2}$$

$$t_ig^q = (t_ig)g^{q-1} \text{ for all } i \tag{3}$$

$$gg^q = g^qg \tag{4}$$

where each of these equations is evaluated in $F/[R,F]$. Equations (1), (3) and (4) yield no restriction on $\overline{x}$ and $\overline{x}_{ji}$. Equation (2) yields that:

$$\overline{x}_{ji} = \overline{x}_{j-1,i-1}\ \overline{x}_{j-1,d}^{-e_i}\ \overline{x}_{i-1,d}^{e_j} \qquad \text{for } 2 \leq j < i \leq d$$
$$\overline{x}_{1i} = \overline{x}_{i-1,d} \qquad \text{for } 2 \leq i \leq d.$$

By induction on j, these are equivalent to the following set of equations:

$$\overline{x}_{ji} = \overline{x}_{1,i-j+1} \prod_{k=2}^{j} \overline{x}_{1k}^{-e_{i-j+k}}\ \overline{x}_{1,i-j+k}^{e_k} \qquad \text{for } 2 \leq j < i \leq d$$
$$\overline{x}_{1i} \prod_{k=2}^{i-1} \overline{x}_{1k}^{e_{d-i+1+k}} = \overline{x}_{1,d-i+2} \prod_{k=2}^{i-1} \overline{x}_{1,d-i+1+k}^{e_k} \qquad \text{for } 2 \leq i \leq d.$$

As $e_i = e_{d-i+2}$ for $2 \leq i \leq d$, by induction on i it follows that these equations are equivalent to:

$$\overline{x}_{ji} = \overline{x}_{1,i-j+1} \prod_{k=2}^{j} \overline{x}_{1k}^{-e_{i-j+k}}\ \overline{x}_{1,i-j+k}^{e_k} \qquad \text{for } 2 \leq j < i \leq d$$
$$\overline{x}_{1i} = \overline{x}_{1,d-i+2} \qquad \text{for } 2 \leq i \leq d.$$

It is now straightforward to read off that the elements $\overline{x}$, $\overline{x}_{0j}$ for $1 \leq j \leq d$ and $\overline{x}_{1i}$ for $2 \leq i \leq d/2+1$ are a free generating set for $R/[R,F]$. Hence $R/[R,F] \cong \mathbb{Z}_p^{1+d+d/2}$. As C_t/C_t' is finite, it follows that $R/(R \cap F') \cong \mathbb{Z}_p^{1+d}$. Thus $M(C_t) \cong (R \cap F')/[R,F] \cong \mathbb{Z}_p^{d/2}$ as desired. ∎

Blackburn [1] determined a formula for the Schur multiplicator of a wreath product $G \wr H$, where G and H are finite groups. This formula shows that $M(G)$ embeds into $M(G \wr H)$ in this case. Blackburn's proof uses the natural presentation of a wreath product and it can be extended to the following result on infinite pro-p-groups.

8 Theorem: *Let G be an infinite pro-p-group of finite coclass and let H be finite. Then $M(G)$ embeds into $M(G \wr H)$.*

Wreath products yield an important construction for infinite pro-p-groups of finite coclass: the groups $W_t(s) = C_t \wr P_{s-t}$, where P_{s-t} is a Sylow p-subgroup of $\mathrm{Sym}(p^{s-t})$, are infinite pro-p-groups of finite coclass with trivial hypercenter and central exponent t. Theorem 8 has the following direct application on the Schur multiplicators of the groups $W_t(s)$, which yields a concrete realisation of the torsion-free rank of $M(W_t(s))$.

9 Corollary: *$M(C_t)$ embeds into $M(W_t(s))$.*

7. Infinite pro-2-groups of finite coclass

We determined the Schur multiplicators of the infinite pro-2-groups of coclass at most 3 using the computer algebra system Gap [8], see Figures 1-3. A list of all 60 infinite pro-2-groups of coclass at most 3 can be found in [5]. Figures 1-3 list these 60 pro-2-groups with their dimension d, their coclass r, their central exponent t and their hypercenter of order p^m, and it exhibits the abelian invariants of the torsion subgroups and the torsion-free ranks of their Schur multiplicators.

Figure 3 also shows that the main Theorem of this paper is not valid for infinite pro-2-groups of finite coclass, as Theorem 6 does not hold for the prime 2; See the groups 40 and 41 in Figure 3.

There are 5 groups with trivial Schur multiplicator among the groups in Figures 1-3. Explicit presentations for these 5 groups are included in the following.

$$G_1 := \langle a, t \mid a^2 = 1, t^a = t^{-1} \rangle$$
$$G_2 := \langle a, t \mid a^4 = 1, t^a = t^{-1} \rangle$$
$$G_3 := \langle a, t \mid a^8 = 1, t^a = t^{-1} \rangle$$
$$G_4 := \langle a, t, b \mid a^2 = b^2, b^4 = 1, t^a = t^{-1}b, b^a = b^{-1}, b^t = b^{-1} \rangle$$
$$G_5 := \langle a, b, c, t_1, t_2, d \mid a^2 = d, b^2 = t_2, c^2 = t_1, d^2 = 1, b^a = c, c^a = b, c^b = ct_1^{-1}t_2d, t_1^a = t_2, t_2^a = t_1, t_1^b = t_1^{-1}, t_2^c = t_2^{-1} \rangle$$

	d	r	t	m	$T(M(G))$	$\mathrm{tf}(M(G))$
1	1	1	1	0	()	0

Figure 1. Infinite pro-2-groups of coclass 1

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	d	r	t	m	$T(M(G))$	$\text{tf}(M(G))$
1	1	2	1	1	(2,2)	0
2	1	2	1	1	(2)	0
3	1	2	1	1	()	0
4	2	2	2	0	()	1
5	2	2	1	0	(2)	0

Figure 2. Infinite pro-2-groups of coclass 2

	d	r	t	m	$T(M(G))$	$\text{tf}(M(G))$
1	1	3	1	2	(2,2,2,2,2)	0
2	1	3	1	2	(2,2,2)	0
3	1	3	1	2	(2,2)	0
4	1	3	1	2	(2,2)	0
5	1	3	1	2	(2,2)	0
6	1	3	1	2	(2,4)	0
7	1	3	1	2	(2,2,2)	0
8	1	3	1	2	(2)	0
9	1	3	1	2	(2)	0
10	1	3	1	2	(4)	0
11	1	3	1	2	(2,2)	0
12	1	3	1	2	(2)	0
13	1	3	1	2	(2)	0
14	1	3	1	2	(2)	0
15	1	3	1	2	(2)	0
16	1	3	1	2	(2)	0
17	1	3	1	2	()	0
18	1	3	1	2	()	0
19	2	3	2	1	(2,2)	1
20	2	3	2	1	(2)	1
21	2	3	2	1	(2)	1
22	2	3	2	1	()	1
23	2	3	2	1	()	1
24	2	3	2	1	()	1
25	2	3	1	1	(2,2,2)	0
26	2	3	1	1	(2,2)	0
27	2	3	1	1	(2,2)	0
28	2	3	1	1	(2)	0
29	2	3	1	1	(2,2)	0
30	2	3	1	1	(2)	0
31	2	3	1	1	(2)	0
32	2	3	1	1	()	0
33	2	3	1	1	(2,2,2)	0
34	4	3	3	0	()	2
35	4	3	2	0	()	1
36	4	3	2	0	(2,2)	1
37	4	3	2	0	(2,2)	1
38	4	3	2	0	(2)	1
39	4	3	1	0	(2,2)	0
40	4	3	1	0	()	1
41	4	3	1	0	(2,2)	1
42	4	3	1	0	(2,2)	0
43	4	3	1	0	(2,2)	0
44	4	3	1	0	(2,2,4)	0
45	4	3	1	0	(2)	0
46	4	3	1	0	(2,2)	0
47	4	3	1	0	(2,2,8)	0
48	4	3	1	0	(2,4)	0
49	4	3	1	0	(2,2,2)	0
50	4	3	1	0	(2,2)	0
51	4	3	1	0	(2,4)	0
52	4	3	1	0	(2,2,2)	0
53	4	3	1	0	(2,2)	0
54	4	3	1	0	()	2

Figure 3. Infinite pro-2-groups of coclass 3

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